MPP–2006–100
hep-ph/0611373

Higher-Order Corrected Higgs Bosons in `FeynHiggs 2.5` *

T. HAHN [1†], S. HEINEMEYER [2‡], W. HOLLIK [1§], H. RZEHAK [3**],
G. WEIGLEIN [4††] AND K. WILLIAMS [4‡‡]

[1] Max-Planck-Institut für Physik (Werner-Heisenberg-Institut),
Föhringer Ring 6, D–80805 Munich, Germany

[2] Instituto de Fisica de Cantabria (CSIC–UC), Santander, Spain

[3] Paul Scherrer Institut, Würenlingen und Villigen, CH–5232 Villigen PSI, Switzerland

[4] IPPP, University of Durham, Durham DH1 3LE, U.K.

Abstract

Large higher-order corrections enter the Higgs boson sector of the MSSM via Higgs-boson self-energies. Their effects have to be taken into account for the correct treatment of loop-corrected Higgs-boson mass eigenstates as external (on-shell) or internal particles in Feynman diagrams. We review how the loop corrections, including momentum dependence and imaginary contributions, are correctly taken into account for external (on-shell) Higgs boson and how effective couplings can be derived. The proceedures are implemented in the code `FeynHiggs 2.5`.

[*] two talks given by S. Heinemeyer at the *LCWS06*, 9-13 March 2006, Bangalore, India
[†] email: hahn@feynarts.de
[‡] email: Sven.Heinemeyer@cern.ch
[§] email: hollik@mppmu.mpg.de
[**] email: Heidi.Rzehak@psi.ch
[††] email: Georg.Weiglein@durham.ac.uk
[‡‡] email: k.e.williams@durham.ac.uk

PRAMANA
— journal of
physics

© Indian Academy of Sciences

pp. 1–8

Higher-Order Corrected Higgs Bosons in FeynHiggs 2.5

T. Hahn[1], S. Heinemeyer[2], W. Hollik[1], H. Rzehak[3], G. Weiglein[4], K. Williams[4]
[1] Max-Planck-Institut für Physik, Föhringer Ring 6, D–80805 Munich, Germany
[2] Instituto de Fisica de Cantabria (CSIC-UC), Santander, Spain
[3] Paul Scherrer Institut, Würenlingen und Villigen, CH–5232 Villigen PSI, Switzerland
[4] IPPP, University of Durham, Durham DH1 3LE, U.K.

Abstract. Large higher-order corrections enter the Higgs boson sector of the MSSM via Higgs-boson self-energies. Their effects have to be taken into account for the correct treatment of loop-corrected Higgs-boson mass eigenstates as external (on-shell) or internal particles in Feynman diagrams. We review how the loop corrections, including momentum dependence and imaginary contributions, are correctly taken into account for external (on-shell) Higgs boson and how effective couplings can be derived. The proceedures are implemented in the code FeynHiggs 2.5.

Keywords. MSSM, Higgs, higher-order corrections

PACS Nos 2.0

1. Introduction

The search for Higgs bosons is a crucial test of Supersymmetry (SUSY) which can be performed with the present and the next generation of accelerators. A precise prediction for the production and decay processes of the Higgs bosons in terms of the relevant SUSY parameters is necessary in order to determine the discovery and exclusion potential of the Tevatron [1–3], and for physics at the LHC [4–7] and the ILC [8–12]. A precise prediction of Higgs-boson properties is also required as input for (current) electroweak precision analyses, see e.g. Refs. [13,14]

Since the soft SUSY-breaking parameters can in general be complex, it is necessary to take into account the effects of complex phases. In the Minimal Supersymmetric Standard Model with complex parameters (cMSSM) Higgs physics is affected by complex parameters entering via loop corrections. In particular, these are the Higgs mixing parameter, μ, the trilinear couplings, A_f, $f = t, b, \tau, \ldots$, and the gaugino mass parameters M_1, M_2, M_3, where $|M_3| \equiv m_{\tilde{g}}$ (the gluino mass). As a consequence, once loop corrections are taken into account the neutral Higgs bosons are no longer $\mathcal{CP}$-eigenstates, but mix with each other [15],

$$(h, H, A) \to h_1, h_2, h_3 \ . \tag{1}$$

As tree-level input parameters in the Higgs sector (besides the gauge couplings and the Z boson mass) it is convenient to choose the mass of the charged Higgs boson, $M_{H^\pm}$, and the ratio of the two vacuum expectation values, $\tan\beta$.

In order to obtain the prediction for the Higgs masses beyond lowest order, the poles of the Higgs propagators have to be determined. Since the propagator poles are located in the complex plane, we define the physical mass of each particle according to the real part of the complex pole.

Neglecting the mixing with the Goldstone bosons and the (longitudinal parts) of the Z boson (see Ref. [16] for details) one can write the propagator matrix of the neutral Higgs bosons h, H, A as a 3×3 matrix, $\Delta_{hHA}(p^2)$. It is related to the 3×3 matrix of the irreducible vertex functions by

$$\Delta_{hHA}(p^2) = -\left(\hat{\Gamma}_{hHA}(p^2)\right)^{-1}, \tag{2}$$

where

$$\hat{\Gamma}_{hHA}(p^2) = \mathrm{i}\left[p^2 \mathbb{1} - \mathbf{M}_\mathrm{n}(p^2)\right], \tag{3}$$

$$\mathbf{M}_\mathrm{n}(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \tag{4}$$

Inversion of $\hat{\Gamma}_{hHA}(p^2)$ yields for the diagonal Higgs propagators ($i = h, H, A$)

$$\Delta_{ii}(p^2) = \frac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\mathrm{eff}}(p^2)}, \tag{5}$$

where $\Delta_{hh}(p^2)$, $\Delta_{HH}(p^2)$, $\Delta_{AA}(p^2)$ are the (11), (22), (33) elements of the 3×3 matrix $\Delta_{hHA}(p^2)$, respectively. The structure of eq. (5) is formally the same as for the case without mixing, but the usual self-energy is replaced by the effective quantity $\hat{\Sigma}_{ii}^{\mathrm{eff}}(p^2)$ which contains mixing contributions of the three Higgs bosons. It reads (no summation over i, j, k)

$$\hat{\Sigma}_{ii}^{\mathrm{eff}}(p^2) = \hat{\Sigma}_{ii}(p^2) - \mathrm{i}\frac{2\hat{\Gamma}_{ij}(p^2)\hat{\Gamma}_{jk}(p^2)\hat{\Gamma}_{ki}(p^2) - \hat{\Gamma}_{ki}^2(p^2)\hat{\Gamma}_{jj}(p^2) - \hat{\Gamma}_{ij}^2(p^2)\hat{\Gamma}_{kk}(p^2)}{\hat{\Gamma}_{jj}(p^2)\hat{\Gamma}_{kk}(p^2) - \hat{\Gamma}_{jk}^2(p^2)}, \tag{6}$$

where the $\hat{\Gamma}_{ij}(p^2)$ are the elements of the 3×3 matrix $\hat{\Gamma}_{hHA}(p^2)$ as specified in eq. (3).

For completeness, we also state the expression for the off-diagonal Higgs propagators. It reads ($i \neq j$, no summation over i, j, k)

$$\Delta_{ij}(p^2) = \frac{\hat{\Gamma}_{ij}\hat{\Gamma}_{kk} - \hat{\Gamma}_{jk}\hat{\Gamma}_{ki}}{\hat{\Gamma}_{ii}\hat{\Gamma}_{jj}\hat{\Gamma}_{kk} + 2\hat{\Gamma}_{ij}\hat{\Gamma}_{jk}\hat{\Gamma}_{ki} - \hat{\Gamma}_{ii}\hat{\Gamma}_{jk}^2 - \hat{\Gamma}_{jj}\hat{\Gamma}_{ki}^2 - \hat{\Gamma}_{kk}\hat{\Gamma}_{ij}^2}, \tag{7}$$

where we have dropped the argument p^2 of the $\hat{\Gamma}_{ij}(p^2)$ appearing on right-hand side for ease of notation.

The complex pole $\mathcal{M}^2$ of each propagator is determined as the solution of

$$\mathcal{M}_i^2 - m_i^2 + \hat{\Sigma}_{ii}^{\mathrm{eff}}(\mathcal{M}_i^2) = 0. \tag{8}$$

Writing the complex pole as

$$\mathcal{M}^2 = M^2 - iM\Gamma, \tag{9}$$

where M is the mass of the particle and Γ its width, and expanding up to first order in Γ around M^2 yields the following equation for M_i^2,

$$M_i^2 - m_i^2 + \mathrm{Re}\hat{\Sigma}_{ii}^{\mathrm{eff}}(M_i^2) + \frac{\mathrm{Im}\,\hat{\Sigma}_{ii}^{\mathrm{eff}}(M_i^2)\,\left(\mathrm{Im}\,\hat{\Sigma}_{ii}^{\mathrm{eff}}\right)'(M_i^2)}{1 + \left(\mathrm{Re}\hat{\Sigma}_{ii}^{\mathrm{eff}}\right)'(M_i^2)} = 0. \tag{10}$$

The short-hand notation $f'(p^2) \equiv d\,f(p^2)/(d\,p^2)$ has been used, and M_i denotes the loop-corrected mass, while m_i is the lowest-order mass ($i = h, H, A$).

While the Higgs-boson masses M_i^2 can in principle directly be determined from eq. (10) by means of an iterative procedure (since M_i^2 appears as argument of the self-energies in eq. (10)), it is often more convenient to determine the mass eigenvalues from a diagonalization of the mass matrix in eq. (4). In the code `FeynHiggs 2.5`we perform a numerical diagonalization of eq. (4) using an iterative Jacobi-type algorithm [17]. The mass eigenvalues M_i are then determined as the zeros of the function $\mu_i^2(p^2) - p^2$, where $\mu_i^2(p^2)$ is the ith eigenvalue of the mass matrix in eq. (4) evaluated at p^2. Insertion of the resulting eigenvalues M_i into eq. (10) verifies (to $\mathcal{O}(\Gamma)$) that each eigenvalue indeed corresponds to the appropriate (complex pole) solution of the propagator. We define the loop-corrected mass eigenvalues according to

$$M_{h_1} \leq M_{h_2} \leq M_{h_3}. \tag{11}$$

In our determination of the Higgs-boson masses we take into account all imaginary parts of the Higgs-boson self-energies (besides the term with imaginary parts appearing explicitly in eq. (10), there are also products of imaginary parts in $\mathrm{Re}\hat{\Sigma}_{ii}^{\mathrm{eff}}(M_i^2)$). The effects of the imaginary parts of the Higgs-boson self-energies on Higgs phenomenology can be especially relevant if the masses are close to each other. This has been analyzed in Ref. [18] taking into account the mixing between the two heavy neutral Higgs bosons, where the complex mass matrix has been diagonalized with a complex mixing angle, resulting in a non-unitary mixing matrix. The effects of imaginary parts of the Higgs-boson self-energies on physical processes with s-channel resonating Higgs bosons are discussed in Refs. [18–20]. In Ref. [18] only the one-loop corrections from the $t/\tilde{t}$ sector have been taken into account for the H–A mixing, analyzing the effects on resonant Higgs production at a photon collider. In Ref. [19] the full one-loop imaginary parts of the self-energies have been evaluated for the mixing of the three neutral MSSM Higgs bosons. The effects have been analyzed for resonant Higgs production at the LHC, the ILC and a photon collider (however, the corresponding effects on the Higgs-boson masses have been neglected). In Ref. [20] the $\tilde{t}/\tilde{b}$ one-loop contributions (neglecting the t/b corrections) on the H–A mixing for resonant Higgs production at a muon collider have been discussed. Our calculation incorporates for the first time the complete effects arising from the imaginary parts of the one-loop self-energies in the neutral Higgs-boson propagator matrix, including their effects on the Higgs masses and the Higgs couplings in a consistent way.

As described above, the solution for the Higgs-boson masses in the general case where the full momentum dependence and all imaginary parts of the Higgs-boson self-energies are taken into account is numerically quite involved. It is therefore of interest to consider also approximate methods for determining the Higgs-boson masses. Instead of keeping the full momentum dependence in eq. (4), the "p^2 on-shell" approximation consists of setting the arguments of the self-energies appearing in eq. (4) to the tree-level masses according to ($i, j = h, H, A$)

$$\begin{aligned} p^2 \text{ on-shell approximation:} \quad & \hat{\Sigma}_{ii}(p^2) \to \hat{\Sigma}_{ii}(m_i^2) \\ & \hat{\Sigma}_{ij}(p^2) \to \hat{\Sigma}_{ij}((m_i^2 + m_j^2)/2) \ . \end{aligned} \tag{12}$$

In this way the Higgs-boson masses can simply be obtained as the eigenvalues of the (momentum-inpendent) matrix of eq. (4). The "p^2 on-shell" approximation has the benefit that it removes all residual dependencies on the field renormalization constants that cannot be avoided in the iterative procedure.

Instead of setting the momentum argument of the renormalized self-energies to the tree-level masses, in the "$p^2 = 0$" approximation the momentum dependence of the self-energies is neglected completely ($i, j = h, H, A$),

$$\begin{aligned} p^2 = 0 \text{ approximation:} \quad & \hat{\Sigma}_{ii}(p^2) \to \hat{\Sigma}_{ii}(0) \\ & \hat{\Sigma}_{ij}(p^2) \to \hat{\Sigma}_{ij}(0) \ . \end{aligned} \tag{13}$$

In the "$p^2 = 0$" approximation the masses are identified with the eigenvalues of $\mathbf{M}_\mathrm{n}(0)$ (see eq. (4)) instead of the true pole masses. This approximation is mainly useful for comparisons with effective-potential calculations. The matrix $\mathbf{M}_\mathrm{n}(0)$ is hermitian by construction.

2. Amplitudes with external Higgs Bosons

In evaluating processes with external (on-shell) Higgs bosons beyond lowest order one has to account for the mixing between the Higgs bosons in order to ensure that the outgoing particle has the correct on-shell properties such that the S matrix is properly normalized. This gives rise to finite wave-function normalization factors.[1] For the case of 2×2 mixing appearing in the MSSM with real parameters (rMSSM) for the mixing between the two neutral $\mathcal{CP}$-even Higgs bosons h and H, which is analogous to the mixing of the photon and Z boson in the Standard Model, the relevant wave function normalization factors are well-known, see e.g. Refs. [23,24]. An amplitude with an external Higgs boson, i, receives the corrections ($i, j = h, H$, no summation over i, j)

$$\sqrt{\hat{Z}_i} \left(\Gamma_i \ + \ \hat{Z}_{ij} \Gamma_j \right) \quad (i \neq j) \ , \tag{14}$$

[1]The introduction of these factors can in principle be avoided by using a renormalization scheme where all involved particles obey on-shell conditions from the start, but it is often more convenient to work in a different scheme like the $\overline{\mathrm{DR}}$ scheme for the field renormalizations [22].

where the $\Gamma_{i,j}$ denote the one-particle irreducible Higgs vertices, and

$$\hat{Z}_i = \left[1 + \mathrm{Re}\hat{\Sigma}'_{ii}(p^2) - \mathrm{Re}\left(\frac{\left(\hat{\Sigma}_{ij}(p^2)\right)^2}{p^2 - m_j^2 + \hat{\Sigma}_{jj}(p^2)} \right)' \right]^{-1}_{\Big|p^2=M_i^2}, \tag{15}$$

$$\hat{Z}_{ij} = -\frac{\hat{\Sigma}_{ij}(M_i^2)}{M_i^2 - m_j^2 + \hat{\Sigma}_{jj}(M_i^2)} . \tag{16}$$

As before m_j denotes the tree-level mass, while M_i is the loop-corrected mass.

In the case of the cMSSM, the formulas above need to be extended to the case of 3×3 mixing. A vertex with an external Higgs boson, i, has the form (with i, j, k all different, $i, j, k = h, H, A$, and no summation over indices)

$$\sqrt{\hat{Z}_i} \left(\Gamma_i \; + \; \hat{Z}_{ij}\Gamma_j \; + \; \hat{Z}_{ik}\Gamma_k + \dots \right) , \tag{17}$$

where the ellipsis represents contributions from the mixing with the Goldstone boson and the Z boson, see Ref. [16] for more details. The finite Z factors are given by

$$\hat{Z}_i = \frac{1}{1 + \left(\mathrm{Re}\hat{\Sigma}^{\mathrm{eff}}_{ii}\right)'(M_i^2)}, \tag{18}$$

$$\begin{aligned} \hat{Z}_{ij} &= \frac{\Delta_{ij}(p^2)}{\Delta_{ii}(p^2)}_{\Big|p^2=M_i^2} \\ &= \frac{\hat{\Sigma}_{ij}(M_i^2)\left(M_i^2 - m_k^2 + \hat{\Sigma}_{kk}(M_i^2)\right) - \hat{\Sigma}_{jk}(M_i^2)\hat{\Sigma}_{ki}(M_i^2)}{\hat{\Sigma}^2_{jk}(M_i^2) - \left(M_i^2 - m_j^2 + \hat{\Sigma}_{jj}(M_i^2)\right)\left(M_i^2 - m_k^2 + \hat{\Sigma}_{kk}(M_i^2)\right)}, \end{aligned} \tag{19}$$

where the propagators $\Delta_{ii}(p^2)$, $\Delta_{ij}(p^2)$ have been given in eqs. (5) and (7), respectively. Using eq. (17) with Z factors specified in eqs. (18), (19) and adding to this expression the mixing contributions of the Higgs bosons with the Goldstone bosons and the gauge bosons (see Ref. [16] for more details) yields the correct normalization of the outgoing Higgs bosons in the S matrix.

For convenience we define a matrix $\tilde{\mathbf{Z}}_{\mathrm{n}}$ based on the wave function normalization factors. Its elements are given by (with $\hat{Z}_{ii} = 1$, $i, j = h, H, A$, and no summation over i)

$$(\tilde{\mathbf{Z}}_{\mathrm{n}})_{ij} := \sqrt{\hat{Z}_i}\, \hat{Z}_{ij} . \tag{20}$$

Performing a re-ordering of the lines of $\tilde{\mathbf{Z}}_{\mathrm{n}}$ such that they correspond to the mass ordering of eq. (11) results in the matrix $\mathbf{Z}_{\mathrm{n}}$. A vertex with an external (on-shell) Higgs boson h_i is then given by

$$(\mathbf{Z}_{\mathrm{n}})_{i1}\Gamma_h + (\mathbf{Z}_{\mathrm{n}})_{i2}\Gamma_H + (\mathbf{Z}_{\mathrm{n}})_{i3}\Gamma_A + \dots \tag{21}$$

where the ellipsis again represents contributions from the mixing with the Goldstone boson and the Z boson.

3. Effective couplings

In a general amplitude with internal Higgs bosons, the structure describing the Higgs part is given by $\sum_{ij} \Gamma_i \; \Delta_{ij} \; \Gamma_j$, where the $\Gamma_{i,j}$ are as above the one-particle irreducible Higgs vertices, and the propagators Δ_{ij} are given in eqs. (5) and (7). For phenomenological analyses it is often convenient to use approximations of improved-Born type with effective couplings incorporating leading higher-order effects. There is no unique prescription how to define such effective coupling terms. One possibility would be to consider the matrix $\mathbf{Z}_\mathrm{n}$, defined through eqs. (20)–(21), as mixing matrix. The elements of the matrix $\mathbf{Z}_\mathrm{n}$, however, are in general complex, so that the $\mathbf{Z}_\mathrm{n}$ is a non-unitary matrix. Therefore it cannot be interpreted as a rotation matrix. If one wants to introduce effective couplings by means of a (unitary) rotation matrix, it is necessary to make further approximations.

A possible choice leading to a unitary rotation matrix is the "$p^2 = 0$" approximation, which is used in the effective potential approach. As before, we first consider the case of 2×2 mixing relevant for the rMSSM. In the "$p^2 = 0$" approximation defined in eq. (13) the momentum dependence in the renormalized self-energies is set to the respective tree-level Higgs boson masses, so that the derivative in eq. (15) acts only on the p^2 term in the propagator factor. In this limit $\hat{Z}_i$ simplifies to [25,26]

$$p^2 = 0 \text{ approximation, } 2 \times 2 \text{ mixing:} \qquad \hat{Z}_i = \frac{1}{1 + \hat{Z}_{ij}^2} \; . \tag{22}$$

For the mixing between the neutral $\mathcal{CP}$-even Higgs bosons h, H this yields $\hat{Z}_h = \hat{Z}_H = \cos^2 \Delta\alpha$. This corresponds to an effective coupling approximation where the tree-level mixing angle α appearing in the couplings of the neutral $\mathcal{CP}$-even Higgs bosons is replaced by $\alpha_\mathrm{eff} = \alpha + \Delta\alpha$ [25,26].

It is easy to verify that for the 3×3 mixing case eq. (18) in the "$p^2 = 0$" approximation simplifies to

$$p^2 = 0 \text{ approximation, } 3 \times 3 \text{ mixing:} \qquad \hat{Z}_i = \frac{1}{1 + \hat{Z}_{ij}^2 + \hat{Z}_{ik}^2} \; , \tag{23}$$

as a direct generalization of eq. (22).

The matrix $\mathbf{Z}_\mathrm{n}$ defined through eqs. (20)–(21) goes over into a unitary matrix $\mathbf{R}_\mathrm{n}$ in this approximation,

$$p^2 = 0 \text{ approximation, } 3 \times 3 \text{ mixing:} \qquad \mathbf{Z}_\mathrm{n} \to \mathbf{R}_\mathrm{n}, \quad \mathbf{R}_\mathrm{n} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} . \tag{24}$$

The matrix $\mathbf{R}_\mathrm{n}$ diagonalizes the matrix $\mathbf{M}_\mathrm{n}(0)$ arising from eq. (4) in the "$p^2 = 0$" approximation. $\mathbf{R}_\mathrm{n}$ can therefore be used to connect the mass eigenstates h_1, h_2, h_3 with the original states h, H, A,

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2=0} = \mathbf{R}_\mathrm{n} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix} , \; \mathbf{R}_\mathrm{n} \, \mathbf{M}_\mathrm{n}(0) \, \mathbf{R}_\mathrm{n}^\dagger = \begin{pmatrix} M^2_{h_1\, p^2=0} & 0 & 0 \\ 0 & M^2_{h_2\, p^2=0} & 0 \\ 0 & 0 & M^2_{h_3\, p^2=0} \end{pmatrix} . \tag{25}$$

Since $\mathbf{M}_\mathrm{n}$ is hermitian, the matrix $\mathbf{R}_\mathrm{n}$ is unitary.

As shown in Ref. [16], a better approximation of the full result can be achieved by defining the effective couplings in the "p^2 on-shell" approximation. The unitary matrix $\mathbf{U}_\mathrm{n}$ is defined such that it diagonalizes the matrix $\mathrm{Re}\big(\mathbf{M}_\mathrm{n}(p^2 \text{ on-shell})\big)$ arising from eq. (4) in the "p^2 on-shell" approximation and neglecting the imaginary parts. This yields

$$p^2 \text{ on-shell approx.:} \quad \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}_{p^2 \text{ on-shell}} = \mathbf{U}_\mathrm{n} \cdot \begin{pmatrix} h \\ H \\ A \end{pmatrix}, \; \mathbf{U}_\mathrm{n} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \quad (26)$$

$$\mathbf{U}_\mathrm{n}\, \mathrm{Re}\big(\mathbf{M}_\mathrm{n}(p^2 \text{ on-shell})\big)\; \mathbf{U}_\mathrm{n}^\dagger = \begin{pmatrix} M^2_{h_1,p^2 \text{ on-shell}} & 0 & 0 \\ 0 & M^2_{h_2,p^2 \text{ on-shell}} & 0 \\ 0 & 0 & M^2_{h_3,p^2 \text{ on-shell}} \end{pmatrix}.$$

The elements of $\mathbf{U}_\mathrm{n}$ can be used to quantify the extent of $\mathcal{CP}$-violation. For example, U_{13}^2 can be understood as the $\mathcal{CP}$-odd part in h_1, while $U_{11}^2 + U_{12}^2$ make up the $\mathcal{CP}$-even part. The unitarity of $\mathbf{U}_\mathrm{n}$ ensures that both parts add up to 1.

The elements of $\mathbf{U}_\mathrm{n}$ can be interpreted as effective couplings of internal Higgs bosons in a loop diagram, which take into account leading higher-order contributions. As an example we show the couplings of a neutral Higgs boson to two gauge bosons, $VV = ZZ, W^+W^-$. Beyond the lowest order in the cMSSM all three neutral Higgs bosons have a $\mathcal{CP}$-even component, so that all three Higgs bosons have non-vanishing couplings to two gauge bosons. The couplings normalized to the SM values are given by

$$g_{h_iVV} = U_{i1} \sin(\beta - \alpha) \; + U_{i2} \cos(\beta - \alpha) \; . \quad (27)$$

4. Conclusions

Large higher-order corrections enter the Higgs boson sector of the MSSM via Higgs-boson self-energies. Their effects have to be taken into account for the correct treatment of loop-corrected Higgs-boson mass eigenstates as external (on-shell) or internal particles in loop diagrams. We have shown how the loop corrections can be taken into account in the treatment of external (on-shell) Higgs bosons, and how effective couplings, taking into account leading higher-order effects, can be constructed.

The proceedure presented here is implemented in the code `FeynHiggs2.5` [16,27–29]. For the Higgs-boson self-energies the complete one-loop corrections [16,30–32] are supplemented by the available two-loop corrections in the Feynman-diagrammatic approach for the MSSM with real parameters [27,28,33] and a resummation of the leading (s)bottom corrections for complex parameters [34]. Higgs boson decays and production cross sections [35] are evaluated using $\mathbf{Z}_\mathrm{n}$ (20,21), ensuring the on-shell properties of the external particles. Effective Higgs boson couplings are obtained with the help of $\mathbf{U}_\mathrm{n}$ (27). In this way `FeynHiggs` provides a precise prediction of Higgs boson production and decay properties and the respective couplings.

References

[1] V. Abazov et al. [D0 Collaboration], hep-ex/0504018.
[2] A. Abulencia et al. [CDF Collaboration], hep-ex/0508051.
[3] [CDF Collaboration], hep-ex/0510065.
[4] ATLAS Collaboration, *Detector and Physics Performance Technical Design Report*, CERN/LHCC/99-15 (1999), see: `atlasinfo.cern.ch/Atlas/GROUPS/PHYSICS/TDR/access.html`.
[5] S. Abdullin et al., *Eur. Phys. J.* **C 39S2** (2005) 41; CMS Physics TDR, see: `cmsinfo.cern.ch/Welcome.html/CMSdocuments/CMSplots/`.
[6] M. Dührssen, S. Heinemeyer, H. Logan, D. Rainwater, G. Weiglein and D. Zeppenfeld, *Phys. Rev.* **D 70** (2004) 113009; hep-ph/0407190.
[7] M. Schumacher, *Czech. J. Phys.* **54** (2004) A103; hep-ph/0410112.
[8] J. Aguilar-Saavedra et al., hep-ph/0106315, see: `tesla.desy.de/tdr`.
[9] T. Abe et al. [American Linear Collider Working Group Collaboration], hep-ex/0106056.
[10] K. Abe et al. [ACFA Linear Collider Working Group Collaboration], hep-ph/0109166, see: `lcdev.kek.jp/RMdraft`.
[11] G. Weiglein et al. [LHC / ILC Study Group], *Phys. Rept.* **426** (2006) 47.
[12] S. Heinemeyer et al., hep-ph/0511332; S. Heinemeyer, hep-ph/0611374.
[13] S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber and G. Weiglein, *JHEP* **0608** (2006) 052; hep-ph/0611371;
[14] J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, *JHEP* **0502** (2005) 013; *JHEP* **0605** (2006) 005; S. Heinemeyer, hep-ph/0611372.
[15] A. Pilaftsis, *Phys. Rev.* **D 58** (1998) 096010; *Phys. Lett.* **B 435** (1998) 88.
[16] M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, hep-ph/0611326.
[17] T. Hahn, physics/0607103.
[18] S. Choi, J. Kalinowski, Y. Liao and P. Zerwas, *Eur. Phys. J.* **C 40** (2005) 555.
[19] J. Ellis, J. Lee and A. Pilaftsis, *Phys. Rev.* **D 70** (2004) 075010; *Nucl. Phys.* **B 718** (2005) 247; *Phys. Rev.* **D 72** (2005) 095006.
[20] J. Bernabeu, D. Binosi and J. Papavassiliou, hep-ph/0604046.
[21] K. Williams et al., *in preparation*.
[22] M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0202166.
[23] A. Dabelstein, *Nucl. Phys.* **B 456** (1995) 25; *Z. Phys.* **C 67** (1995) 495.
[24] T. Hahn, S. Heinemeyer and G. Weiglein, *Nucl. Phys.* **B 652** (2003) 229.
[25] S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. Jour.* **C 16** (2000) 139.
[26] S. Heinemeyer, W. Hollik, J. Rosiek, and G. Weiglein, *Eur. Phys. J.* **C 19** (2001) 535.
[27] S. Heinemeyer, W. Hollik and G. Weiglein, *Eur. Phys. Jour.* **C 9** (1999) 343.
[28] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, *Eur. Phys. J.* **C 28** (2003) 133.
[29] S. Heinemeyer, W. Hollik and G. Weiglein, *Comp. Phys. Comm.* **124** (2000) 76; T. Hahn, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0507009; see `www.feynhiggs.de`.
[30] S. Heinemeyer, *Eur. Phys. Jour.* **C 22** (2001) 521.
[31] M. Frank, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0212037.
[32] S. Heinemeyer, *Int. J. Mod. Phys.* **A 21** (2006) 2659.
[33] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rev.* **D 58** (1998) 091701, *Phys. Lett.* **B 440** (1998) 296.
[34] M. Carena, D. Garcia, U. Nierste and C. Wagner, *Nucl. Phys.* **B 577** (2000) 577.
[35] T. Hahn, F. Maltoni, S. Heinemeyer, G. Weiglein and S. Willenbrock, hep-ph/0607308.